# Vectors 101

A vector is a mathematical quantity that has both magnitude and direction. It is insufficient to describe a vector with only one of the two aspects. A vector is represented symbolically as a directed line segment…or an arrow.

The length of the arrow represents the magnitude of the vector while the angle with respect to the positive *x*-axis represents the direction. Angle representations are usually the same as those you are familiar with from analysis ( are positive *x*, positive *y*, negative *x*, negative *y*, and positive *x*, respectively)

Occasionally, NSEW notation is used to indicate direction. For example, a vector may lie in the NNW direction, read “North-north-west”. This means the direction is halfway between North and Northwest, while Northwest is halfway between North and West.

NNE

ENE

ESE

SSE

SSW

WSW

WNW

NNW

N

E

S

W

NE

NW

SW

SE

Similar Vectors

Two vectors are equal if and only if both the magnitude and direction are the same. If either is different, the vectors are not equal.

 **A B C D**

Only vectors **A** and **C** are equal. **A** and **B** have the same magnitude, but different directions. **A** and **D** have the same direction, but differ in magnitude.

(Note: In typed text, vector names are bold face. In handwriting, an arrow is written above the letter to indicate a vector. Ex: vs. **A**.)

## Opposite of a Vector

The opposite of a vector, written as , means the vector with the same magnitude as **A**, but whose direction differs by .

### -A

### A

## Scalar Multiple of a Vector

In contrast to a vector, a scalar is a mathematical quantity that has magnitude only. (These are just the ordinary real numbers that we know and love!)

A scalar times a vector, written as *a***A**, means the vector with the same direction as **A**, but whose magnitude is *a*A. (The magnitude only of a vector is usually indicated by the same symbol as the vector without the arrow or bold face notation.)

### A

### A

### 2A

## Vector Addition (graphically)

There are two methods to add vectors graphically. The vector sum is called a resultant. The parallelogram method can only add two vectors at a time, while the triangle method can add any number of vectors.

Parallelogram method: a) Place the two vectors to be added with their tails at the origin of a coordinate plane. b) Construct parallel lines to each vector that pass through the tip of the other vector. c) Draw the resultant from the origin to the intersection of these two lines. d) Use a ruler and protractor to measure the magnitude and direction of the resultant.

### A

### B

### A+B

### A+B+C

### C

### A

### B

### A+B

Triangle method: a) Place the first vector with its tail at the origin. b) Place the second vector with its tail at the tip of the first vector. c) If there is a third, fourth, fifth etc… vector, continue with this tail to tip procedure. d) Draw the resultant from the origin to the tip of the last vector. e) Use a ruler and protractor to measure the magnitude and direction of the resultant.

(Notice that **A**+**B** is identical for both the parallelogram and the triangle methods.)

## Vector Components

For any vector, **A**, there are two perpendicular vectors which can be added such that the resultant is **A**. One vector lies in the same direction as the *x* axis and is called the *x* component. The other lies in the same direction as the *y* axis and is called the *y* component.

Notice that **Ax** + **Ay** = **A** as a result of the triangle method. Although Ax and Ay are vectors, they are frequently written as magnitudes only. A positive component magnitude indicates a direction along the positive axis, while a negative component magnitude indicates a direction along the negative axis.

### A

### Ay

### Ax

## Conversion from Magnitude/Direction to Components

If the magnitude of **A** is given by A, and the direction is given by , where  is the angle between the vector **A** and the positive *x* axis, then the components Ax and Ay are given by the following.

 

## Conversion from Components to Magnitude/Direction

If the components Ax and Ay are known, then the magnitude and direction of **A** can be calculated as follows.

 

 

When determining the direction, the sign of Ax will be positive if the vector lies in the first or fourth quadrant, and negative if the vector lies in the second or third. You must remember that the Arctan function is only defined for the first and fourth quadrants. Therefore, **if Ax is negative, you must add 180°** to the calculated  in order to find the actual direction of the vector. (You may also choose to add  to vectors in the fourth quadrant to obtain directions ranging from .)

Adding Vectors Analytically

Adding vectors analytically is always preferred to adding graphically because the resultant is more accurate. But you should always visualize the addition as if you were using a graphical method. As with the triangle method, the analytical method can be used with any number of vectors.

1. Find the *x* and *y* components of each vector to be added.
2. Add the *x* components to find the *x* component of the resultant.
3. Add the *y* components to find the *y* component of the resultant.
4. Using the *x* and *y* components, find the magnitude and direction of the resultant.

## Unit vector notation

There are three unit vectors, (Read “i-hat”, “j-hat”, and “k-hat”)  is the vector whose magnitude is one and is directed along the positive *x* axis.  is the vector whose magnitude is one and is directed along the positive *y* axis. has a magnitude of one and is directed along the positive *z* axis. Any vector can be expressed using these unit vectors.

 

If a vector is two dimensional, the *z* term becomes zero.

## Vector Multiplication I – The Dot Product or Scalar Product

There are two ways to multiply a vector by another vector. One results in a vector, the other results in a scalar. The first we will encounter is the scalar product. There are two ways to calculate the scalar or dot product. If you have magnitude and direction information for your two vectors, then



where *φ* is the smaller angle between **A** and **B.**

If component information is given, then the dot product can be evaluated by multiplying the corresponding components and adding the results.

*φ*

### A

### B

 

## Vector Multiplication II- The Cross Product or Vector Product

There are also two ways to calculate the cross product and the result is always a vector that is perpendicular to both of the vectors being crossed.

If magnitude and direction information is given, the magnitude of the cross product is given by

 

The direction of the cross product is given by the Right Hand Rule. To use the RHR, you first um…obviously… use your right hand. Let your fingers point in the direction of **A**, and wrap your fingers (as if you’re making a fist) through the angle φ towards **B**. Your thumb now points in the direction of the cross product.

*φ*

### A

### B

For this example, the cross product would point into the page.

If component information is given the cross product can be calculated by evaluating the following determinant.

 

The result will be a vector in unit vector notation. If the vectors to be crossed are only two dimensional, then let Az = Bz = 0.

## Examples

Vectors: Displacement Scalars: Distance

 Velocity Speed

 Acceleration Time

 Force Mass

 Pressure Temperature

 Electric Field Charge

 Magnetic Field Volume

 Torque Energy

 Power

 Potential